COMBINING EARTH ORIENTATION MEASUREMENTS USING A KALMAN FILTER

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Combining Earth Orientation Measurements Using a Kalman Filter

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Abstract. A Kalman filter has many properties that make it an attractive choice as a technique for combining Earth orientation measurements. It allows the full accuracy of the measurements to be used, whether the measurements are degenerate or are of full rank, are irregularly or regularly spaced in time, or are corrupted by systematic or other errors that can be described by stochastic models. Proxy Earth orientation measurements can also be used as long as a stochastic model describing the difference between the proxy and "true" Earth orientation parameter can be determined. And by use of a model for the process, a Kalman filter can objectively account for the growth in the uncertainty of the Earth orientation parameters between measurements.

Introduction

The uncertainty in our knowledge of the Earth's changing orientation in space is a major source of error in tracking and navigating interplanetary spacecraft. Because the Earth's orientation changes rapidly and unpredictably, measurements must be acquired frequently and processed rapidly in order to meet the near-real-time Earth orientation requirements of the spacecraft navigation teams. These requirements are currently met by using the Global Positioning System (GPS) to provide daily determinations of polar motion and length-of-day within 24 hours of acquisition. Single baseline Very Long Baseline Interferometry (VLBI) measurements are taken twice-per-month by the Time and Earth Motion Precision Observations (TEMPO) project in order to provide the benchmark Universal Time (UT) measurements between which the GPS length-of-day measurements are integrated. The Kalman Earth

Orientation Filter (KEOF) is then used to combine the GPS polar motion and length-of-day measurements with the TEMPO VLBI variation-of-latitude and UT0 measurements, along with other publicly available Earth orientation measurements, in order to generate and deliver the required polar motion and UT1 Earth orientation parameters to the spacecraft navigation teams.

For more than a decade, the Kalman Earth Orientation Filter has been used at the Jet Propulsion Laboratory (JPL) to combine and predict Earth orientation parameters in support of interplanetary spacecraft tracking and navigation. Kalman filters are commonly used for estimating parameters of some system when a stochastic model of the system is available and when the data contain noise (e.g., Nahi 1969; Gelb 1974; Bierman 1977). For the purpose of combining Earth orientation series, the system consists of a series of the usual Universal Time and polar motion (UTPM) parameters, their excitations, and full covariance matrices. The data consist of series of observed Earth orientation parameters, which may be incomplete and/or degenerate, along with the data measurement covariance matrices. Hernquist et al. (1984) and Eubanks et al. (1985) have described a Kalman filter and smoother developed at JPL for use with UT1 measurements. The approach currently taken at JPL evolved from the earlier UT1 Kalman filter and now estimates all three UTPM parameters, along with their excitations, wherein the polar motion parameters are treated as individual real-valued quantities, rather than as a single complex-valued quantity (Morabito et al. 1988).

The particular design of the JPL Kalman Earth Orientation Filter was dictated to a large extent by the nature of the Earth orientation series being combined, and in particular by the presence of degenerate data types (single station lunar laser ranging and single baseline VLBI observations). Degenerate data types are not measurements of the usual UTPM parameters, but rather are measurements of various linear combinations of UT1 and polar motion, such as UT0 and variation-of-latitude. A Kalman filter enables the inclusion of such degenerate measurements by incorporating into it the linear relationship between the measured degenerate and the usual UTPM parameters.

A Kalman filter has a number of additional useful properties that make it an attractive choice as a means of combining independent Earth orientation data sets. Changes in the Earth's orientation can be described as a randomly excited stochastic process. Consequently, between successive measurements of the Earth orientation parameters, the uncertainty in the knowledge of their values grows and rapidly becomes much larger than the uncertainty in the measurements. Thus, it is important to analyze each measurement at its measurement epoch, rather than at some "nearby" regularized epoch as is commonly done in normal-point methods of combining data sets. Kalman filters are an effective means of dealing with irregularly spaced data sets since the state vector and state covariance matrix can be propagated to the measurement epoch regardless of whether or not the measurements are equi-spaced.

Due to this growth in the uncertainty of the Earth orientation parameters between measurements, when intercomparing data sets in order to evaluate their relative accuracies, it is important to compare independent measurements whose epochs are as close as possible to each other. This argues for comparing an individual data set against a combination of all other independent data sets (rather than against some other individual series) so that the difference in the epochs of the measurements being compared is minimized. Since it is unlikely that independent measurements will be given at exactly the same epoch, it is important that the interpolation procedure used in generating the combined series accounts for the growth in the uncertainty of the Earth orientation parameters between measurements. The JPL Kalman Earth Orientation Filter does this in an objective manner by employing realistic stochastic models of the uncertainty growth between measurements (Morabito et al. 1988).

Finally, in order to obtain the best possible combined series, the degree of smoothing that is applied to the measurements must vary with both the precision and the sampling interval of the measurements. As improvements have been made to the measurement systems (in both hardware and software), the precision with which the measurements have been made has dramatically improved, as has their time resolution. With a Kalman filter, the degree of smoothing applied is a function of both the precision of the measurements and the time span over which the state vector and covariance matrix must be propagated (i.e., of the time resolution of the measurements). (Of course, the Kalman filter model for the growth in the uncertainty of the Earth orientation parameters between measurements is also important in this regard.) Thus, with a Kalman filter, no arbitrary changes in the applied degree of smoothing need be made. The degree of smoothing is automatically adjusted as the precision and time resolution of the measurements change.

Because of these considerations, a Kalman filter-based approach to combining Earth orientation series has been taken at JPL. In this report, the Kalman Earth Orientation Filter is described including a discussion of the characteristics of the Earth orientation measurements being combined, the ability of KEOF to combine measurements having these characteristics, the use of proxy data types, and the stochastic models incorporated into KEOF.

Earth Orientation Measurements

The only space-geodetic measurement technique capable of independently determining all of the Earth orientation parameters (EOPs) is multibaseline VLBI. All of the other techniques need to either apply external constraints to the determined Earth orientation parameters or can determine only subsets of the EOPs, only linear combinations of the EOPs, or only their time rates-of-change. Any method used to combine Earth orientation measurements must therefore be able to combine not only the usual UTPM parameters but must also be able to incorporate subsets, linear combinations, and time derivatives of them. An advantage of KEOF is that not only does it combine such a disparate set of Earth orientation measurements but in the process it makes full use of the inherent accuracies of the measurements provided by each technique.

Single Baseline VLBI

Only two components of the Earth's orientation can be determined from VLBI observations taken on a particular single baseline. A rotation of the Earth about an axis parallel to the baseline connecting the two radio telescopes does not change the relative position of the telescopes with respect to the sources, and hence this component of the Earth's orientation is not determinable from VLBI observations taken on that single baseline. The two components that are determinable are the transverse T component representing a right-handed rotation of the Earth about an axis parallel to the baseline vertical direction, and the vertical V component representing a left-handed rotation of the Earth about an axis parallel to the baseline transverse direction (e.g., Gross et al. 1998, pp. 219–221).

The orthogonal transformation matrix that transforms between the usual UTPM components of the Earth's orientation and the T and V components that are determinable from single baseline VLBI observations can be readily obtained from the coordinates of the radio telescopes located at either end of the baseline (Gross et al. 1998, pp. 219–221). With this transformation matrix KEOF can combine the single baseline VLBI measurements of T and V with other EOP measurements as it routinely does with the single baseline TEMPO VLBI measurements that are processed at JPL from observations acquired using the radio telescopes of the Deep Space Network (DSN) complexes in California, Spain, and Australia. Since KEOF can use the T and V components themselves, there is no need to artificially impose constraints during the data reduction procedure in order to artificially produce UT1 from single baseline VLBI observations, as is done with the National Earth Orientation Service (NEOS) Intensive UT1 series (Robertson et al. 1985; Eubanks et al. 1999). These constraints, which require the use of external polar motion values, can degrade the quality of the resulting Earth orientation determinations. Instead, by being able to use the T and V components themselves, KEOF is able to use the full accuracy without degradation of both components of the Earth's orientation that are determinable from single baseline VLBI observations.

Lunar Laser Ranging

From the single station lunar laser ranging (LLR) technique, two linear combinations of the usual UTPM parameters can be determined, namely, UT0 and the variation of latitude (VOL) at that station (e.g., Mulholland 1980; Lambeck 1988, Chap. 7). These two Earth orientation parameters, UT0 and VOL, are related to UT1 and the polar motion parameters PMX and PMY by the well-known expressions (e.g., Moritz and Mueller 1988, p. 425):

$$\Delta\phi_i(t) \;=\; x_p(t)\cos\lambda_i \;-\; y_p(t)\sin\lambda_i \tag{1a}$$

$$UT0_i(t) - TAI(t) \;=\; U(t) \;+\; x_p(t)\sin\lambda_i\tan\phi_i \;+\; y_p(t)\cos\lambda_i\tan\phi_i \tag{1b}$$

where $\Delta\phi_i$ is the variation of latitude observed at the station i that is located at nominal latitude ϕ_i and east longitude λ_i, the observed UT0 at that station is designated $UT0_i(t) - TAI(t)$ where $TAI(t)$ is a reference time scale based upon atomic clocks, the variable $U(t)$ is defined by $U(t) \equiv UT1(t) - TAI(t)$, and x_p and y_p are the polar motion parameters PMX and PMY, respectively, with y_p being positive towards 90° W longitude. A third linear combination $D_i(t)$ of the UTPM parameters (PMX, PMY, UT1), representing that component of the Earth's orientation that cannot be determined from LLR observations at the single station i, is given by:

$$D_i(t) \;=\; U(t)\sin\phi_i \;-\; x_p(t)\sin\lambda_i\cos\phi_i \;-\; y_p(t)\cos\lambda_i\cos\phi_i \tag{1c}$$

This third, degenerate, component of the Earth's orientation represents changes in the orientation of the Earth resulting from a rotation about the position vector of the station.

As shown by Gross et al. (1998, p. 218), Eq. (1) can be used to define an orthogonal transformation matrix that KEOF uses to transform between the measured UT0, VOL values and the corresponding UTPM values. By means of this transformation matrix, KEOF is able to combine the LLR measurements with other EOP measurements without needing to, for example, first convert the UT0 measurement to UT1 via Eq. (1b), thereby requiring the use of external polar motion values which may be inconsistent with the LLR measurements and hence may degrade the quality of the resulting UT1 value. Instead, KEOF can use without degradation the full accuracy of the LLR UT0 and VOL measurements, their uncertainties, and correlations (Gross et al. 1998, p. 218).

Satellite Laser Ranging

The satellite laser ranging (SLR) technique provides measurements of polar motion and UT1 (e.g., Lambeck 1988, Chap. 6). However, the SLR UT1 measurements are not independently determined since their long period behavior has been constrained to be the same as that of the a priori series (e.g., Eanes and Watkins 1995). Due to the effect of unmodeled forces acting on the satellite, variations in UT1 cannot be separated from variations in the orbital node of the LAGEOS satellite without making additional assumptions (e.g., Lambeck 1988, Sect. 6.3.5). Solutions for these quantities are obtained by assuming that the effects of the unmodeled forces are such that they cause the orbital node to vary slowly, and that rapid variations therefore reflect UT1 behavior. Hence, the slow UT1 variations are not adjusted, but are constrained to be the same as those of the a priori series. Due to software limitations, KEOF does not currently have the capability of separating the measured rapid UT1 variations from the constrained slow variations. Thus, to insure that only independent measurements are used by KEOF, the SLR UT1 values are not currently used. The proper incorporation of the rapid SLR UT1 variations (but not the slow variations) is currently under study.

Global Positioning System

The Global Positioning System can be used to determine polar motion and the time rate-of-change of UT1, or length-of-day (e.g., Bock and Leppard 1990; Blewitt 1993; Hofmann-Wellenhof 1993; Beutler et al. 1996). However, the GPS length-of-day (LOD) measurements are corrupted by unmodeled motion of the GPS satellite constellation. Consequently, the GPS-measured LOD values should be considered as proxy measurements of the Earth's length-of-day since they are not unbiased LOD determinations but include non-negligible time-dependent effects of the motion of the GPS satellite constellation. Thus, before combining the GPS-measured LOD values with other, uncorrupted EOP measurements, the effect of the motion of the satellite constellation on the GPS-measured LOD values must be taken into account. KEOF does this by including a stochastic model of these effects which was determined by analyzing the difference between the proxy GPS LOD measurements and uncorrupted determinations of the Earth's LOD as given by an independent reference series.

Within KEOF, the difference between the GPS measurements $G(t)$ of LOD and "true" LOD $\Lambda(t)$ is modeled as the sum of a random walk and a first order autoregressive (AR1) process:

$$G(t) = \Lambda(t) + \mu_{grw}(t) + \mu_{gar1}(t) \tag{2a}$$

where $\mu_{grw}(t)$ represents a random walk:

$$\frac{d\mu_{grw}(t)}{dt} = \omega_{grw}(t) \tag{2b}$$

and $\mu_{gar1}(t)$ represents an AR1 process:

$$\frac{d\mu_{gar1}(t)}{dt} = -\frac{\mu_{gar1}(t)}{\tau_g} + \omega_{gar1}(t) \tag{2c}$$

and where both $\omega_{grw}(t)$ and $\omega_{gar1}(t)$ represent zero-mean white noise stochastic processes. The decay time τ_g of the AR1 process in Eq. (2c) is set to 2.27 days, and the power spectral densities of the white noise processes in Eqs. (2b) and (2c) are set to 5 μs^2/day and 500 μs^2/day, respectively. This stochastic model for the GPS LOD measurements was empirically determined for JPL's Rapid Service GPS LOD series and thus may not be valid for some other GPS LOD series determined at some other GPS analysis center.

Atmospheric Angular Momentum

Numerous studies have shown that on time scales of a few days to a few years changes in the Earth's length-of-day are dominantly caused by changes in the angular momentum of the atmospheric winds (e.g., Hide and Dickey 1991; Eubanks 1993; Rosen 1993). Determinations of changes in the atmospheric angular momentum (AAM) can therefore be used as proxy length-of-day measurements as long as the relatively small difference between the AAM and LOD is taken into account. As is done for the proxy GPS LOD measurements, KEOF does this by including a stochastic model of this difference (Freedman et al. 1994).

Including AAM in EOP combinations is particularly useful when generating EOP combinations in near-real-time since the AAM values are very timely, being available from the National Centers for Environmental Prediction (NCEP) within a day of data acquisition. Furthermore, as part of NCEP's efforts to predict the weather, forecasts of the AAM are available out to 10 days into the future. These AAM forecast values can be treated as proxy LOD predictions as long as the difference between the AAM forecasts and the LOD is taken into account. Freedman et al. (1994) discuss the use of AAM forecasts by KEOF, showing the marked increase in the accuracy of short-term UT1 predictions when AAM forecasts are included.

Summary

A Kalman filter has many properties that make it an attractive choice as a technique for combining Earth orientation measurements. Since the uncertainty in the values of the Earth orientation parameters grows rapidly in the absence of measurements, intercomparing and combining data sets should be done by a technique that objectively accounts for this uncertainty growth. The Kalman filter is such a technique since it contains a model for the process, and in the absence of measurements uses this model to propagate forward in time the state vector and its covariance matrix. Since the state vector and covariance matrix can be propagated to an arbitrary epoch, the measurements being combined do not need to be equally spaced, nor do measurements determined from different techniques need to be given at the same epoch. Furthermore, when comparing a given series of measurements to a reference series for the purpose of determining adjustments to its stated uncertainties, interpolation error can be minimized by generating the reference series using a Kalman filter that prints its estimates at the epochs of the measurements whose uncertainties are being adjusted (Gross et al. 1998, pp. 223–224).

Since a Kalman filter can include stochastic models for the process, proxy Earth orientation measurements can be used as long as a stochastic model describing the difference between the proxy and "true" Earth orientation parameter can be determined. This ability of the Kalman filter allows systematic errors to be modeled such as those corrupting the GPS LOD measurements, and has allowed KEOF to use AAM analysis and forecast measurements which have dramatically improved the accuracy of short-term UT1 predictions (Freedman et al. 1994).

The use of the information matrix in the Kalman filter allows the incorporation of EOP measurements that span arbitrary subspaces of UTPM parameter space, allowing the use of degenerate data types and not just the usual polar motion and UT1 parameters. This is possible because the information matrix is zero for that subspace of parameter space not spanned by the degenerate measurements (Gross et al. 1998). Thus, the full accuracy of single baseline VLBI measurements and single station LLR measurements can be used when using a Kalman filter to combine them with other EOP measurements.

Finally, when using a Kalman filter to combine independent data sets, the time-varying nature of the precision and temporal resolution of the measurements is automatically taken into account, thereby circumventing the need for any subjective decision to be made as to how or when to change the degree of smoothing applied as the properties of the measured series change. In addition, the correlations between measurements are taken into account with a Kalman filter when the measurement vector is added to the propagated state vector by a vector-weighted average (Gross et al. 1998, pp. 222–223).

In closing, it should be emphasized that the Kalman filter technique allows the full accuracy of the EOP measurements to be used, whether the measurements are degenerate or are of full rank, are irregularly or regularly spaced in time, or are corrupted by systematic or other errors that can be described by stochastic models. Analysis centers should continue to produce the most accurate determinations of the Earth orientation parameters that they possibly can and should not be persuaded to intentionally degrade the accuracy of their product to satisfy a specialized subset of the user community by, for example, artificially constraining their series to be regularly spaced in time or to provide UT1 determinations from single baseline VLBI observations. Techniques such as the Kalman filter are available that allow the full accuracy of the measurements to be utilized for specialized purposes such as EOP intercomparisons and combinations. Members of the broader user community are best served by having access to the most accurate measurements possible, and these should continue to be provided to them by the analysis centers.

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References

Beutler G, Hein GW, Melbourne WG, Seeber G (eds) (1996) GPS Trends in Precise Terrestrial, Airborne, and Spaceborne Applications. Springer-Verlag, New York

Bierman GJ (1977) Factorization Methods for Discrete Sequential Estimation. Academic Press, New York

Blewitt G (1993): Advances in Global Positioning System technology for geodynamics investigations: 1978–1992. In: Smith DE, Turcotte DL (eds), Contributions of Space Geodesy to Geodynamics: Technology, Geodyn. Ser., Vol. 25, AGU, Washington DC, pp 195–213

Bock Y, Leppard N (eds) (1990) Global Positioning System: An Overview. Springer-Verlag, New York

Eanes RJ, Watkins MM (1995): Earth orientation and site coordinates from the Center for Space Research solution. In: Charlot P (ed), Earth Orientation, Reference Frames, and Atmospheric Excitation Functions Submitted for the 1994 IERS Annual Report, IERS Tech. Note 19, Obs. de Paris, Paris, pp L7–L11

Eubanks TM (1993): Variations in the orientation of the Earth. In: Smith DE, Turcotte DL (eds) Contributions of Space Geodesy to Geodynamics: Earth Dynamics, Geodyn Ser 24, AGU, Washington DC, pp 1–54

Eubanks TM, Steppe JA, Spieth MA (1985): The accuracy of radio interferometric measurements of Earth rotation. In: Posner E (ed), The Telecommunications and Data Acquisition Progress Report 42–80: October–December 1984, Jet Propulsion Lab., Pasadena Calif., pp 229–235

Eubanks TM, Archinal BA, Josties FJ, Ray JR (1999): Earth orientation analysis from the U.S. Naval Observatory VLBI program. In: Vandenberg NR (ed), International VLBI Service for Geodesy and Astrometry—1999 Annual Report, NASA/TP–1999-209243, Goddard Space Flight Center, Greenbelt, Md., pp 236–240

Freedman AP, Steppe JA, Dickey JO, Eubanks TM, Sung L-Y (1994): The short-term prediction of universal time and length of day using atmospheric angular momentum. J Geophys Res 99: 6981–6996

Gelb A (ed) (1974) Applied Optimal Estimation. MIT Press, Cambridge Mass

Gross RS, Eubanks TM, Steppe JA, Freedman AP, Dickey JO, Runge TF (1998): A Kalman-filter-based approach to combining independent Earth-orientation series. J Geodesy 72: 215–235

Hernquist DC, Eubanks TM, Steppe JA (1984): A Kalman filter and smoother for Earth rotation (abstract). Spring Meeting Suppl., EOS Trans AGU 65(16): 187

Hide R, Dickey JO (1991): Earth's variable rotation. Science 253: 629–637

Hofmann-Wellenhof B (1993) Global Positioning System: Theory and Practice (2nd edn). Springer-Verlag, New York

Lambeck K (1988) Geophysical Geodesy: The Slow Deformations of the Earth. Oxford University Press, Oxford

Morabito DD, Eubanks TM, Steppe JA, (1988): Kalman filtering of Earth orientation changes. In: Babcock A, Wilkins, GA (eds), The Earth's Rotation and Reference Frames for Geodesy and Geodynamics, D. Reidel, Dordrecht Holland, pp 257–267

Moritz H, Mueller II (1988) Earth Rotation: Theory and Observation. Ungar, New York

Mulholland JD (1980): Scientific advances from ten years of lunar laser ranging. Rev Geophys Space Phys 18: 549–564

Nahi NE (1969) Estimation Theory and Applications. John Wiley and Sons, New York

Robertson DS, Carter WE, Campbell J, Schuh H (1985): Daily Earth rotation determinations from IRIS very long baseline interferometry. Nature 316: 424–427

Rosen RD (1993): The axial momentum balance of Earth and its fluid envelope. Surveys Geophys 14: 1–29